GMAT CLUB

Quant Flash Cards

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Resized for better viewing/printing - jb2010

To memorize

$2^2 = 4$	$11^2 = 121$	$3^3 = 27$
$2^3 = 8$	$12^2 = 144$	$3^4 = 81$
$2^4 = 16$	$13^2 = 169$	$3^5 = 243$
$2^5 = 32$	$14^2 = 196$	$4^3 = 64$
$2^6 = 64$	$15^2 = 225$	$4^4 = 256$
$2^7 = 128$	$16^2 = 256$	$5^3 = 125$
$2^8 = 256$	$17^2 = 289$	$6^3 = 216$
$2^9 = 512$	$18^2 = 324$	$\sqrt{2} = 1.41$
$2^{10} = 1024$	$19^2 = 361$	$\sqrt{3} = 1.73$

Types of numbers

- Natural numbers: The numbers 1,2,3,4…. are called natural numbers or positive integers
- Whole numbers: The numbers 0,1,2,3… are called whole numbers. Whole numbers include "0" and the entire set of natural numbers
- Integers: The numbers ……,-3,-2,-1,0,1,2,3…are called integers. Integers can be positive, negative or "0".
- The numbers 1,2 ,3…. are called positive integers
- The numbers -1 ,-2 ,-3….are called negative integers. '0' is neither positive not negative
- Rational numbers: Any number which is a positive or a negative integer or a fraction, or zero is called a rational number. A rational number is one which can be expressed in the form $\frac{a}{b}$, where b $\neq$0 and a &b are positive or negative integers. E.g. $\frac{1}{3}, \frac{25}{100}$

Divisibility

- **An integer is divisible by**
- 2 if the integer is even
  E.g 2,4,6,8,…
- 3 if the sum of integer's digits is divisible by 3
  72 is divisible by 3 because the sum of its digits is 9, which is divisible by 3. But 103 is not divisible by 3, because the sum of its digits is 4
- 4 if the integer is divisible by 2 twice, or if the last two digits are divisible by 4
  32 is divisible by 4 because it is divisible by 2 twice. For larger numbers, check only the last two digits. For example, 456788 is divisible by 4 because 88 is divisible by 4.
- 5 if the integer ends in 0 or 5
  60 and 65 are divisible by 5, but 61 and 77 are not.
- 6 if the integer is divisible by both 2 and 3
  42 is divisible by 6 since it is divisible by 2 and by 3.
- 8 if the integer is divisible by 2 three times, or if the last three digits are divisible by 8
  72 is divisible by 8 since it is divisible by 2 once (36), twice (18), and a third time (9). For larger numbers, check only the last 3 digits. For example, 786816 is divisible by 8 because 816 is divisible by 8.
- 9 if the sum of the integer's digits is divisible by 9.
  810 is divisible by 9 since the sum of its digits is 9, which is divisible by 9
- 10 if the integer ends in 0.
- 11 if the difference between the sum of the digits in the odd place and the sum of the digits in the even place of the number is either 0 or a multiple of 11.
  Example 121, Sum of digits in odd place = 1+1 =2 and
  Sum of the digits in even place =2. The difference is 2-2=0. So, 121 is divisible by 11.
  Another example, 1859, Sum of digits in odd place = 8+9 =17 and sum of digits in even place=5+1=6. Diff=17-6=11.
- 12 if the integer is divisible by both 3 and 4.

Prime Numbers

- A prime number is a natural number with exactly two distinct natural number divisors: 1 and itself
- 1 is not a prime number
- Verifying primality can be done by trial division. The simplest trial division method tests whether n is a multiple of an integer m between 2 and $\sqrt{n}$
- For example, 541. $\sqrt{541}$ is between 23 and 24. Test if any of the prime numbers less than 23 is a factor of 541. The prime numbers 2,3,5,7,11,13,17,23 and hence 541 is a prime number
- *Example GMAT problem,*
- *if p is a prime number greater than 2, what is the value of p ?*
- *(1) There are a total of 100 prime numbers between 1 and p+1*
  (2) There are a total of p prime numbers between 1 and 3,912
- *1: p has to be 100th prime number starting from 2 (1st prime number) sufficient.*
  2: all the prime numbers between 1 and 3912 can be found and the number of prime number will be what we need. Sufficient.
  Hence D.
- *For more explanations, please refer to http://gmatclub.com/forum/ds-prime-numbers-91947.html*

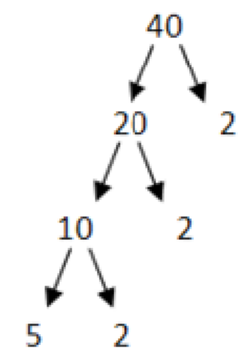
All the prime numbers under 100
2,3,5,7,11,13,17,23,29,31,37,41,43,47,53,59,61,67,71,73,79,83,89,97

Factorization

- The **prime factors** of a positive integer are the prime numbers that divide that integer exactly, without leaving a remainder
- For instance, integer n with three unique prime factors can be expressed as n= $a^p * b^q * c^r$
- So, 40 can be expressed as $2^3 * 5^1$
- Number of factors for a given number N can be expressed by
  (p+1)(q+1)(r+1) where p,q and r are prime factors of N
  Applying this formula for 40 will give (3+1)*(1+1)=8 factors
- The number of ways of expressing a given number as a product of two factors is given by
  1/2 (p+1)(q+1)(r+1) for non perfect squares
  1/2 {(p+1)(q+1)(r+1)+1} for perfect squares
- All natural numbers have an even number of factors except 1 and perfect squares
- Sum of all factors of a number will be equal to

$$\frac{a^{p+1}-1}{a-1} * \frac{b^{q+1}-1}{b-1} * \frac{c^{r-1}-1}{c-1} \ldots\ldots$$

- E.g., 40 $\frac{2^4-1}{2-1} * \frac{5^2-1}{5-1} = 15 * 9 = 90$



Small	Large
1	40
2	20
4	10
5	8

GCF and LCM

- GCF
  - List prime factorization for each number
  - Extract overlap
- LCM
  - List prime factorization for each number
  - Extract highest power for every prime factor

Number	2	3	5	7	
27		3^3			
84	2^2	3^1		7^1	
90	2^1	3^2	5^1		
GCF		3^1			$= 3^1$
LCM	2^2	3^3	5^1	7^1	$= 2^2 * 3^3 * 5^1 * 7^1$=3780

GCF and LCM of fractions

$$LCM of fractions = \frac{LCM - of - numerators}{GCF - of - denaminators}$$

$$LCM - of \frac{3}{4} and \frac{1}{2} = \frac{LCM - of - 3 and 1}{GCF - of - 4 and 2} = \frac{3}{2}$$

$$GCF - of - fractions = \frac{GCF - of - numerators}{LCM - of - denominators}$$

$$GCF - of \frac{3}{4} and \frac{1}{2} = \frac{GCF - of - 3 and 1}{LCM - of - 4 and 2} = \frac{1}{4}$$

Remainder Theory

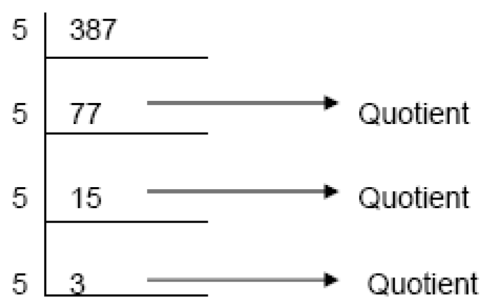
- A remainder is defined as the integer portion of the dividend that is not evenly divisible by the divisor
- $\frac{x}{N} = Q + \frac{R}{N} \longleftarrow$ Remainder $\qquad \frac{23}{4} = 5 + \frac{3}{4} \longleftarrow$ Remainder
- Dividend = Quotient * Divisor + Remainder
- E.g., 23 =5*4 +3
- You can add and subtract remainders directly, as long as you correct excess or negative remainders.
- You can multiply remainders, as long as you correct excess remainders at the end.
- Remainder theorem states that when f(x), a polynomial function in x is divided by (x-a), the remainder is f(a).
- Finding remainder in divisions involving powers of numbers
- Remainder of the division of the form $\frac{2^{56}}{7}$

- **Pattern method**
- Remainder when 2^1 is divided by 7 is 2
- Remainder when 2^2 is divided by 7 is 4
- Remainder when 2^3 is divided by 7 is 1
- Remainder when 2^4 is divided by 7 is 2
- We find that the remainder repeats in 4th step
- If we divide 56 is two more than multiple of 3
- Will have the remainder when 2^2 is divided by 7. so the remainder is 4

- **Remainder Theorem method**
- $\frac{2^2 * (2^3)^{18}}{2^3 - 1}$ Here x= 2^3 and a=1
- So remainder, as per remainder theorem, is $2^2 * (1)^{18}$ = 4

Factorial

- Factorial is defined for any positive integer. It is denoted by !. Thus factorial of n is written as n!
- n! is defined as the product of all the integers from 1 to n
- So, n!= 1 * 2 * 3 * ……(n-1) * n
- 0! = 1 and 1! =1
- **Finding the largest power of a number contained in the factorial of a given number**
- What is the largest power of 5 contained in 387!
-



- As 3 can not be divided by 5, we stop here and add all the quotients 77+15+3 = 95. Hence, 95 is the largest power of 5 that can divide 387! Without leaving any remainder

Factorial contd

- **Trailing zeros:**
  Trailing zeros are a sequence of 0's in the decimal representation (or more generally, in any positional representation) of a number, after which no other digits follow.

  125000 has 3 trailing zeros;

  The number of trailing zeros in the decimal representation of **n!**, the factorial of a non-negative integer , can be determined with this formula: $\frac{n}{5}+\frac{n}{5^2}+\frac{n}{5^3}+...+\frac{n}{5^k}$, where k must be chosen such that . $5^k<n$

  For example:

  How many zeros are in the end (after which no other digits follow) of 32! ?
  (denominator must be less than 32, $5^2=25$ is less)

  Hence, there are 7 zeros in the end of 32!

  The formula actually counts the number of factors 5 in n!, but since there are at least as many factors of 2, this is equivalent to the number of factors 10, each of which gives one more trailing zero.

  Finding the number of powers of a prime number P , in the n!.

  The formula is: $\frac{n}{p}+\frac{n}{p^2}+\frac{n}{p^3}$ ... till $p^x<n$

  What is the power of 2 in 25!?

  $$\frac{25}{2}+\frac{25}{4}+\frac{25}{8}+\frac{25}{16}=12+6+3+1=22$$

  2 | 25
  2 | 12 → Quotient
  2 | 6 → Quotient
  2 | 3 → Quotient
  2 | 1 → Quotient

  By adding quotients method also we get same the same value.

Factorial contd

- **Finding the power of non-prime in n!:**

  How many powers of 900 are in 50!

  Make the prime factorization of the number: $900 = 2^2 * 3^2 * 5^2$, then find the powers of these prime numbers in the n!.
- Find the power of 2:
  $\frac{50}{2}+\frac{50}{4}+\frac{50}{8}+\frac{50}{16}+\frac{50}{32}=25+12+6+3+1=47$

  $= 2^{47}$
- Find the power of 3:
  $\frac{50}{3}+\frac{50}{9}+\frac{50}{27}=16+5+1=22$

  $= 3^{22}$
- Find the power of 5:
  $\frac{50}{5}+\frac{50}{25}=10+2=12$

  $= 5^{12}$
- We need all the prime {2,3,5} to be represented twice in 900, 5 can provide us with only 6 pairs, thus there is 900 in the power of 6 in 50!.

Odds and Evens

Odd $\pm$ Even	ODD
Odd $\pm$ Odd	EVEN
Even $\pm$ Even	EVEN

Odd * Odd	ODD
Even * Even	EVEN
Odd * Even	EVEN

- Even numbers can be represented as 2n
- Odd numbers can be represented as 2n+1 or 2n-1, where n is an integer.

Exponents

- Things to remember
- Even exponent hides the sign of the base, for eg.,3^2
- An exponential expression with a base of 0 always yields 0, regardless of the exponent.
- An exponential expression with a base of 1 always yields 1, regardless of the exponent.
- An exponential expression with a base of -1 yields 1 when the exponent is even, and yields -1 when the exponent is odd

Rules	Example
$x^m * x^n = x^{m+n}$	$x^2 * x^3 = x^5$
$x^0 = 1$	$(2xyz)^0 = 1$
$\frac{x^m}{x^n} = x^{m-n}$	$\frac{y^7}{y^3} = y^4$
$(x^m)^n = x^{mn}$	$(y^4)^3 = y^{12}$
$(xy)^m = x^m * y^m$	$(xy)^3 = x^3 * y^3$
$(\frac{x}{y})^n = \frac{x^n}{y^n}$	$(\frac{x}{y})^3 = \frac{x^3}{y^3}$
$x^m * y^m = (xy)^m$	$2^3 * 3^3 = (2*3)^3 = 6^3$

Roots

1. $\sqrt{x} * \sqrt{x} = x$

2. $\sqrt{x} * \sqrt{y} = \sqrt{xy}$

3. $(\sqrt{x} + \sqrt{y})^2 = x + y + 2\sqrt{xy}$

4. $(\sqrt{x} - \sqrt{y})^2 = x + y - 2\sqrt{xy}$

5. $(\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y}) = x - y$

6. $\dfrac{1}{\sqrt{x}} = \dfrac{1}{\sqrt{x}} * \dfrac{\sqrt{x}}{\sqrt{x}}$

7. $\dfrac{x}{\sqrt{x}} = \dfrac{x}{\sqrt{x}} * \dfrac{\sqrt{x}}{\sqrt{x}} = \dfrac{x\sqrt{x}}{x} = \sqrt{x}$

8. $\dfrac{1}{\sqrt{x} + \sqrt{y}} = \dfrac{1}{\sqrt{x} + \sqrt{y}} * \dfrac{\sqrt{x} - \sqrt{y}}{\sqrt{x} - \sqrt{y}} = \dfrac{\sqrt{x} - \sqrt{y}}{x - y}$

9. $\dfrac{1}{\sqrt{x} - \sqrt{y}} = \dfrac{1}{\sqrt{x} - \sqrt{y}} * \dfrac{\sqrt{x} + \sqrt{y}}{\sqrt{x} + \sqrt{y}} = \dfrac{\sqrt{x} + \sqrt{y}}{x - y}$

10. $\dfrac{\sqrt{x} - \sqrt{y}}{\sqrt{x} + \sqrt{y}} = \dfrac{\sqrt{x} - \sqrt{y}}{\sqrt{x} + \sqrt{y}} * \dfrac{\sqrt{x} - \sqrt{y}}{\sqrt{x} - \sqrt{y}} = \dfrac{(\sqrt{x} - \sqrt{y})^2}{x - y} = \dfrac{x + y - 2\sqrt{xy}}{x - y}$

11. $\dfrac{\sqrt{x} + \sqrt{y}}{\sqrt{x} - \sqrt{y}} = \dfrac{\sqrt{x} + \sqrt{y}}{\sqrt{x} - \sqrt{y}} * \dfrac{\sqrt{x} + \sqrt{y}}{\sqrt{x} + \sqrt{y}} = \dfrac{(\sqrt{x} + \sqrt{y})^2}{x - y} = \dfrac{x + y + 2\sqrt{xy}}{x - y}$

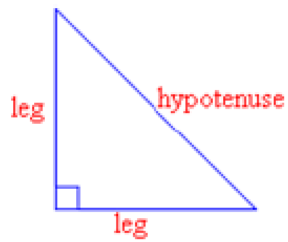
12. $(\sqrt{a} + \sqrt{b})(\sqrt{c} - \sqrt{d}) + (\sqrt{a} - \sqrt{b})(\sqrt{c} + \sqrt{b}) = 2(\sqrt{ac} - \sqrt{bd})$

PEMDAS

- **Order of operations**
- P – Parentheses
- E – Exponents
- M&D – Multiplication and division
- A&S – Addition and Subtraction

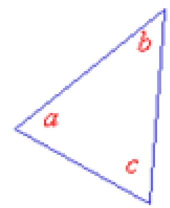
Triangles

- **Different types of triangles**



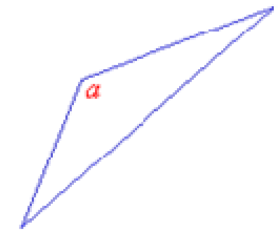
Right Triangle

One of the angles is 90°



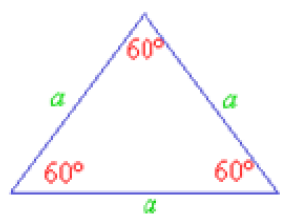
Acute Triangle

All angles are < 90°



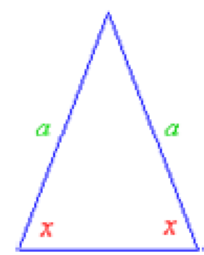
Obtuse Triangle

One of the angles > 90°



Equilateral Triangle

All the angles are 60°



Isosceles Triangle

Two angles and sides are equal

Area of a triangle

Area of a triangle = ½ bh

Perimeter of a triangle = a+b+c

Semiperimeter $s = \frac{a+b+c}{2}$

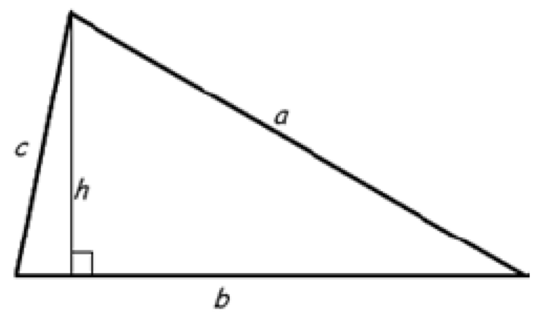
Then $\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$

Area can be arrived by = r * S = abc/4R

Where r – inradius
S – semiperimeter
R - circumradius

Triangle

Perimeter $p = a + b + c$



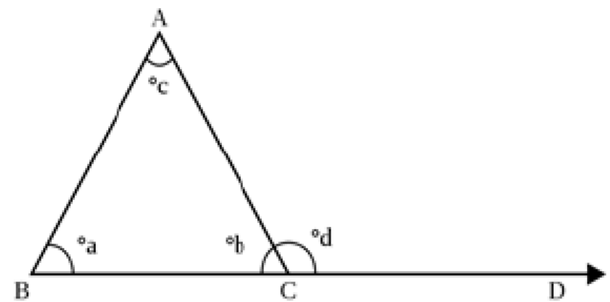
Area $A = \frac{bh}{2}$ or $A = \frac{1}{2} bh$

- Triangle inequality: The sum of the lengths of any two sides of a triangle always exceeds the length of the third side
    - Eg., a+b > c
- Length of third side must lie between the difference and sum of the two given sides. Eg., c<a+b and c>a-b

Angles of a triangle

- Sum of three angles of a triangle equals 180^0
- In the given picture $a+b+c = 180^0$
- Angles correspond to their opposite sides
  - This means that the largest angle is opposite of longest side
  - If two sides are equal, their opposite angles are also equal
- An exterior angle of a triangle is an angle that is a linear pair (and hence supplementary) to an interior angle.

  So, $d = a+b$



- Pythagorean Theorem

  $a^2+b^2 = c^2$

  $3^2+4^2 = 5^2$

- Common Right Triangles

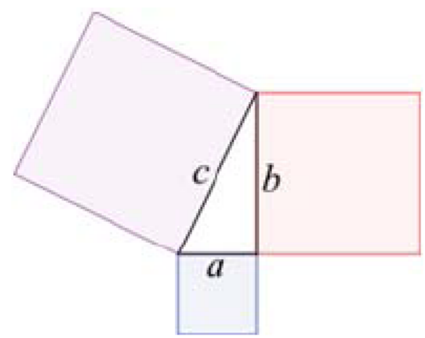
  3:4:5

  5:12:13

  8:15:17

  7:24:25

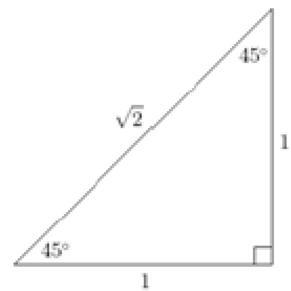
  9:40:41



Isosceles and equilateral triangles

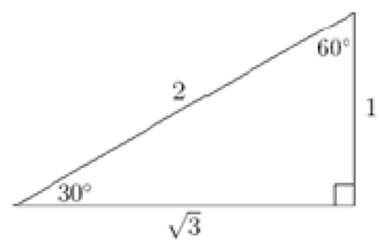
- Isosceles triangle has two sides equal

$1 : 1 : \sqrt{2}$



- 30-60-90 Triangle

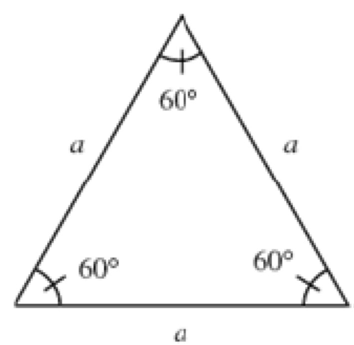
$1 : \sqrt{3} : 2$



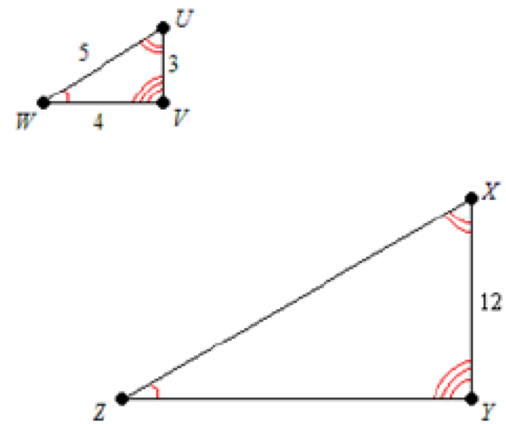
- Equilateral Triangle

$$A = a^2 \frac{\sqrt{3}}{4}$$

$$h = a \frac{\sqrt{3}}{2}$$

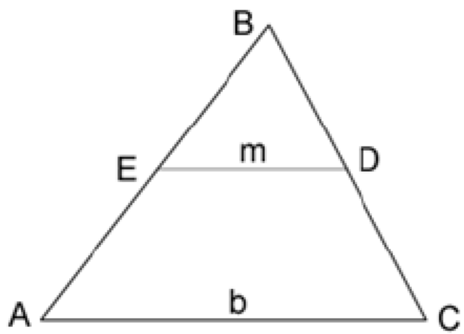


Similar Triangles



- Have the same shape but not the same size
- Each corresponding angles are equal
- Ratio of any pair of corresponding sides is the same

Midsegment of a Triangle A line segment joining the midpoints of two sides of a triangle



$$m = \frac{1}{2}b$$

- A triangle has 3 possible midsegments.
- **The midsegment is always parallel to the third side of the triangle.**
- **The midsegment is always half the length of the third side.**
- A triangle has three possible midsegments, depending on which pair of sides is initially joined.

Polygons

- Polygon Formulas
(N = # of sides and S = length from center to a corner)
- **Area** of a regular polygon = (1/2) N sin(360°/N) S2
- **Sum** of the interior angles of a polygon = (N - 2) x 180°
- The **number of diagonals** in a polygon = 1/2 N(N-3)
- The **number of triangles** (when you draw all the diagonals from one vertex) in a polygon = (N - 2)

Polygon Names
Generally accepted names

Sides	**Name**
n	N-gon
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
10	Decagon
12	Dodecagon

Polygons and interior angles

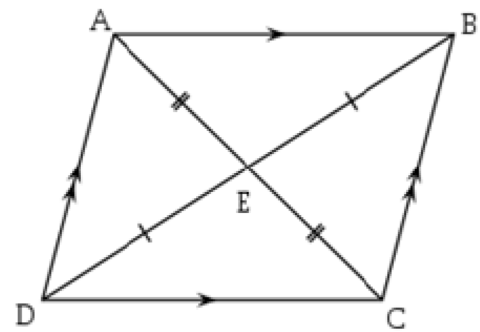
			If it is a **Regular Polygon...**	
Shape	**Sides**	**Sum of Internal Angles**	**Shape**	**Each Angle**
Triangle	3	180°		60°
Quadrilateral	4	360°		90°
Pentagon	5	540°		108°
Hexagon	6	720°		120°
Heptagon *(or Septagon)*	7	900°		128.57...°
Octagon	8	1080°		135°
...	...	..	...	...
Any Polygon	**n**	(n-2) × 180°		(n-2) × 180° / **n**

Polygons

Parallelogram

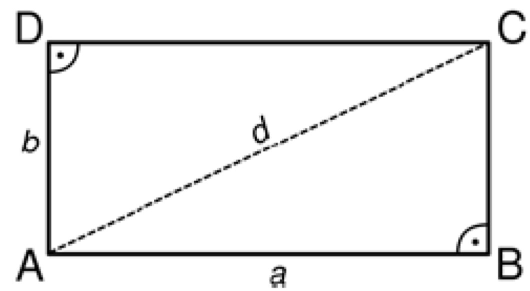
- Opposite sides are equal in length
- Opposite angles are equal in measure
- Opposite side will never intersect
- Diagonals of a parallelogram bisect each other

- Perimeter = 2(a+b) where a and b are adjacent sides
- Area = b * h



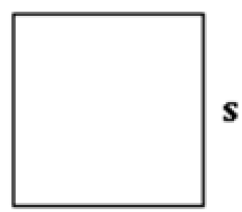
Rectangle

- Opposite sides are parallel and congruent
- The diagonals are equal in length and bisect each other
- Its sides meet at right angle (90^0).
- Area = a * b
- Perimeter = 2(a+b)
- Diagonal = $\sqrt{a^2 + b^2}$



Square

- Area = s^2
- Perimeter = 4s
- Length of diagonal = $s\sqrt{2}$
- Area using diagonal = $\frac{d^2}{2}$

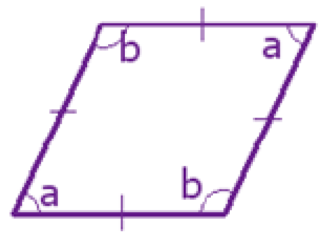


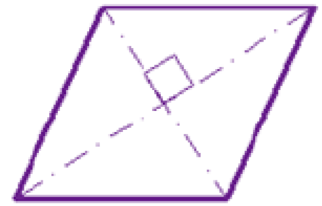
Polygons

Rhombus

- Four sided shape where all four sides have equal length
- Opposite angles are equal in measure
- Opposite side are parallel
- Diagonals of bisect each other at right angles

- Perimeter = 4b
- Area = b * h
- Area = $\frac{d1*d2}{2}$



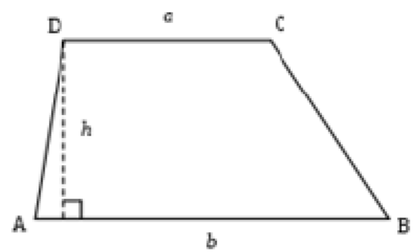


Trapezoid

- Area $A = \frac{a+b}{2} \cdot h$
- The mid-segment of a trapezoid is the segment that joins the midpoints(m) of the non-parallel sides.

$$m = \frac{a+b}{2}$$

- Then area = m * h

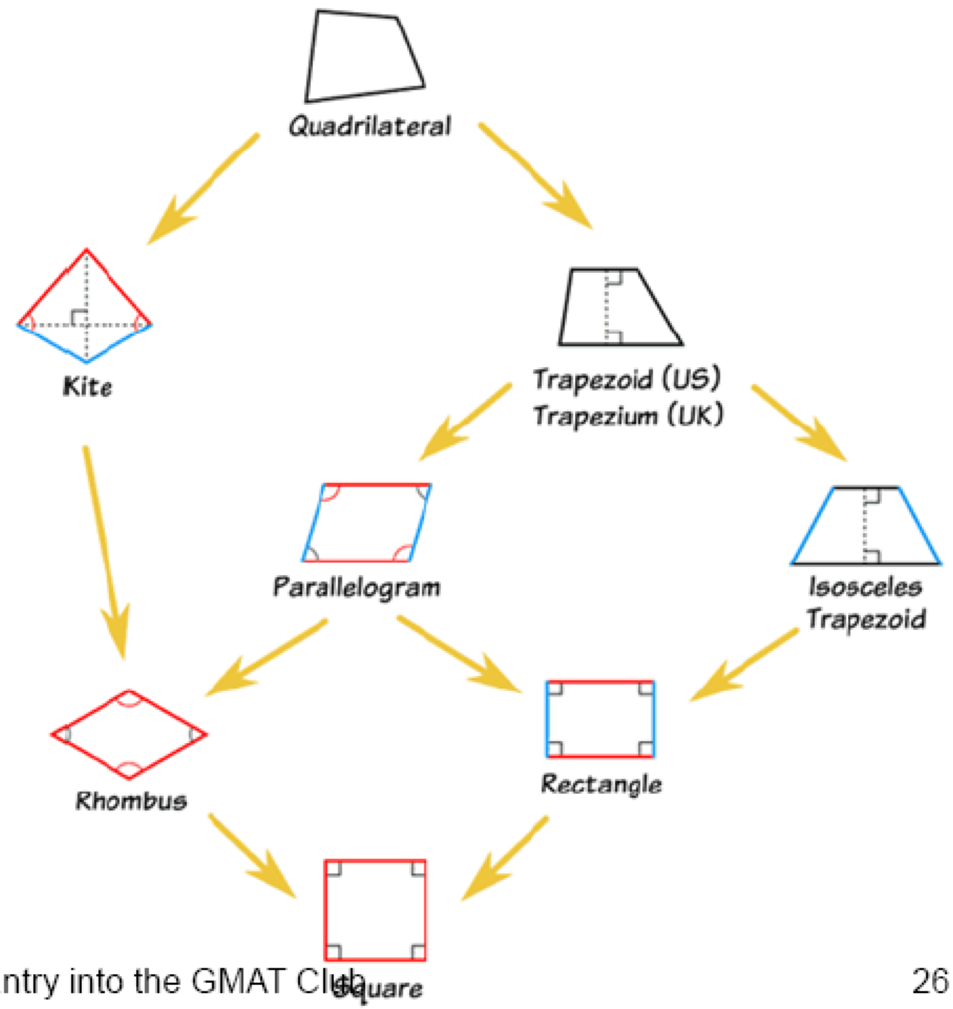


Regular Hexagon

- Perimeter=6* side
- Area = $\frac{3\sqrt{3}}{2}s^2$

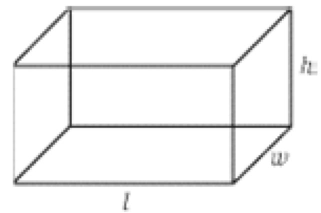
Polygons family chart

- Using the chart below you can answer such questions as:
- Is a Square a type of Rectangle? (Yes)
- Is a Rectangle a type of Kite? (No)

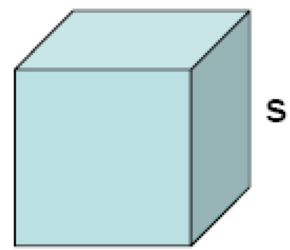


3D

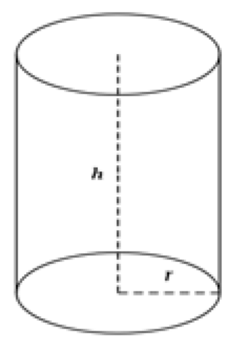
- **Rectangular solid**
- Volume = l*b*w
- Surface area = 2(lb+lw+bw)
- Space diagonal =$\sqrt{l^2 + b^2 + w^2}$



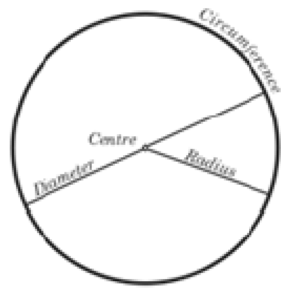
- **Cube**
- Surface area= 6s^2
- Volume = s^3
- Space diagonal = $s\sqrt{3}$



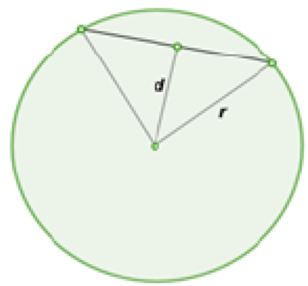
- **Sphere**
- Surface area = 2 circles + rectangle = $A = 2\pi r^2 + 2\pi rh = 2\pi r(r + h).$
- Volume $V = \pi r^2 h$



Circles



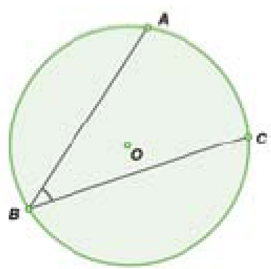
- Circumference of circle $c = 2\pi r$
- Area of a circle $A = \pi r^2$



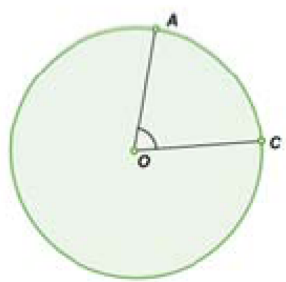
- Length of a chord $Length = 2\sqrt{r^2 - d^2}$
- Length of an arc $2\pi r * \dfrac{\theta}{360}$
- Area of a sector

$$A = \pi r^2 \cdot \frac{\theta}{360}$$

- Inscribed vs. Central Angles



Inscribed angle

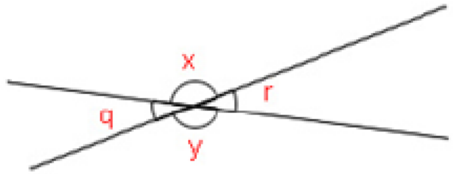


Central angle

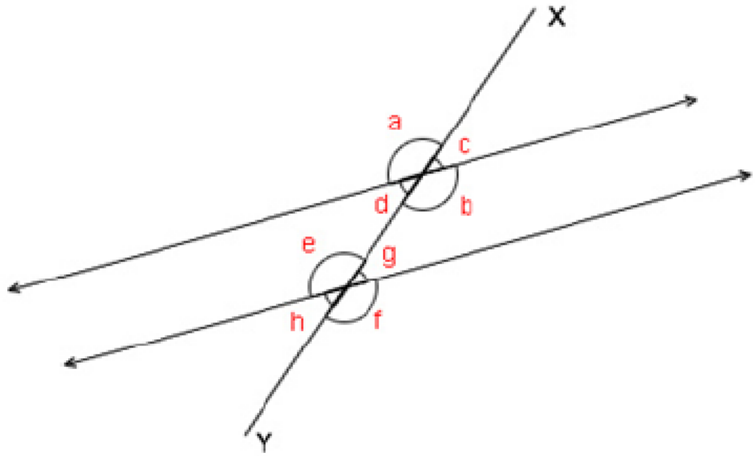
The Central Angle Theorem states that the measure of inscribed angle is always half the measure of the central angle.

Lines and angles

- Interior angles formed by intersecting lines form a circle
- So, x+y+q+r = 360
- Interior angles that combine to form a line sum to 180
- So, x+r= 180
- Angles found opposite to each other are equal
- So, x=y and q=r



- This figure has 8 angles but there are only two unique angles
- All angles <90 are equal
- All angles >90 are also equal
- So, a=b=e=f and c=d=g=h

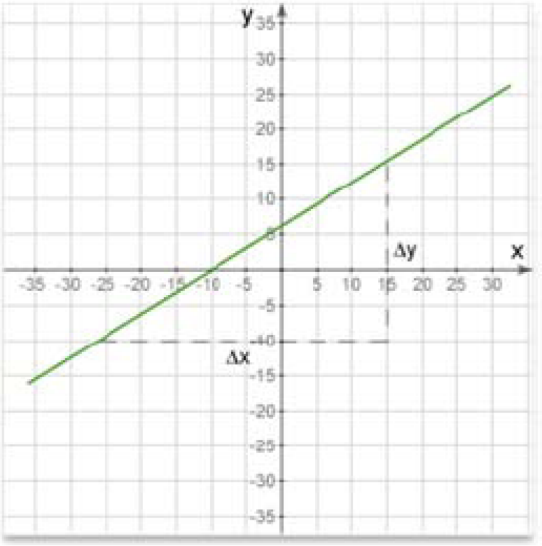


Slope and distance

Slope

- The slope of the line passing through the points $P1 = (x_1, y_1)$ and $P2 = (x_2, y_2)$ is

$$m = \frac{rise}{run} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

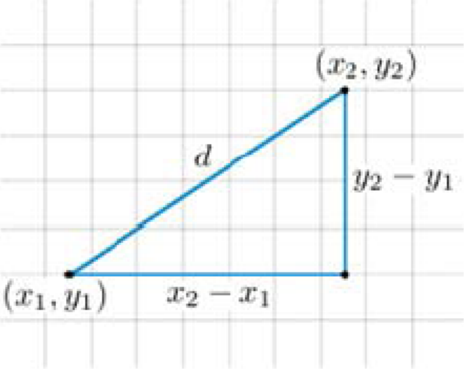


Slope direction

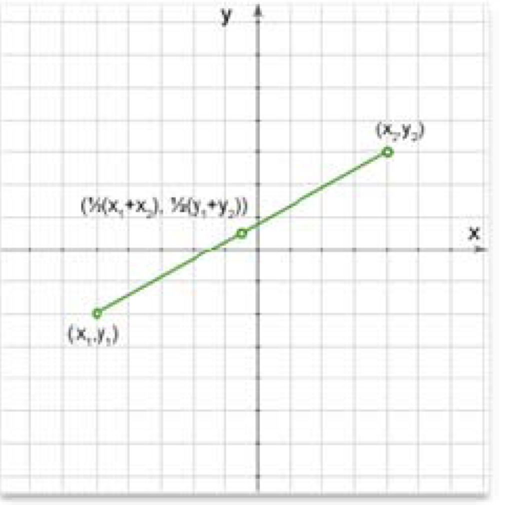
The slope of a line can be positive, negative, zero or undefined.

Distance between two points

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2},$$



Midpoint of a line segment



The x-coordinate of the midpoint is the average of the x-coordinates of the two endpoints.

$$x_m = \frac{x_1 + x_2}{2}$$

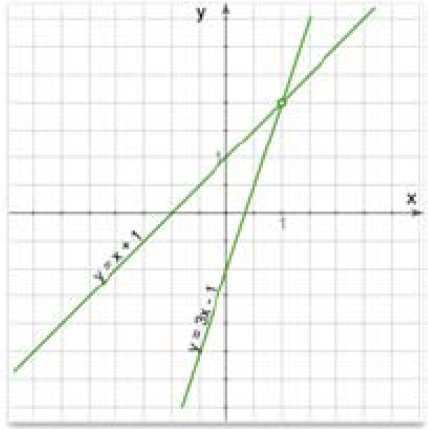
• Likewise, the y-coordinate is the average of the y-coordinates of the endpoints.

$$y_m = \frac{y_1 + y_2}{2}$$

Lines in coordinate geometry

- Equation of line passing through (x_1, y_1) and (x_2, y_2) is represented by the formula
  $$\frac{y - y_1}{y_1 - y_2} = \frac{x - x_1}{x_1 - x_2}$$
- Equation of line if the slope m and y intercept c of a line are given
- y=mx+c
- Equation of line if one point the line pass and the slope is known
  $$y - y_1 = m(x - x_1)$$
- Equation of line if x intercept a and y intercept b is given
  $$\frac{x}{a} + \frac{y}{b} = 1$$

Intersection of two straight lines



To find the intersection of two straight lines:

1. First we need their equations
2. Then, since at the point of intersection, the two equations will share a point and thus have the same values of x and y, we set the two equations equal to each other. This gives an equation that we can solve for x
3. We substitute the x value in one of the line equations (it doesn't matter which) and solve it for y.

This gives us the x and y coordinates of the intersection.

Example #1

Q: Find the point of intersection of two lines that have the following equations (in slope-intercept form):

$$y = 3x - 3$$
$$y = 2.3x + 4$$

Solution: At the point of intersection they will both have the same y-coordinate value, so we set the equations equal to each other:

$$3x - 3 = 2.3x + 4$$

This gives us an equation in one unknown (x) which we can solve:

$$x = 10$$

To find y, simply set x equal to 10 in the equation of either line and solve for y:

Equation for a line $y = 3x - 3$ (Either line will do)

Set x equal to 10: $y = 30 - 3$

$y = 27$

We now have both x and y, so the intersection point is (10, 27)

Translations

x **has** three-fifths **as many** wins as x	$x = \frac{3}{5}y$
x **has** three times **as many** losses as y	x = 3y
x **percent of** y	$\frac{x}{100}y$
x is **decreased by** y **percent**	$x * (1 - \frac{y}{100})$
x **is increased by** y **percent**	$x * (1 + \frac{y}{100})$
x is **increased by <u>a factor of</u>** 10	10 * x
y **percent less than** x. Means x minus y percent of x	$x * (1 - \frac{y}{100})$
y **percent more than** x	$x * (1 + \frac{y}{100})$

Rates

- **Traveling at different rates:**
- A bus traveling at an average rate of 50 kilometers per hour made the trip to town in 6 hours. If it had traveled at 45 kilometers per hour, how many more minutes would it have taken to make the trip?
- Solution:
  Step 1: Set up a *rtd* table.
- Step 2: Fill in the table with information given in the question.
- A bus traveling at an average rate of 50 kilometers per hour made the trip to town in 6 hours. If it had traveled at 45 kilometers per hour, how many more minutes would it have taken to make the trip?
- Let t = time to make the trip in Case 2.
- Step 3: Fill in the values for d using the formula $d = rt$
- Step 4: since both traveled the same distance
  45t= 300=> t=40 mins

	r	t	d
Case1			
Case2			

	r	t	d
Case1	50	6	50*6=300
Case2	45	t	45t

Work

- **Rates are Additive: 1/Tx + 1/Ty = 1/Txy**
- **Rate * Time = Work**
- **Rate = 1/(Time to Complete); Time to Complete = 1/Rate**
- **Tips**
- Try to solve using rate additivity principle; only use the RT = W equation if necessary.
- Organize information in a data table.
- Columns: Rate, Time, Work
- Rows: Case 1, Case 2, etc.
- **Consider plugging in numbers of you're dealing with a work problem with a lot of theoretical unknowns** (e.g., where a lot of variables are expressed in terms of percentages and/or ratios).
- **"Same Work" Problems are Simply Inverse Proportionality Problems**
- Explanation: When the amount of work is held constant, you only need to work with rate and time. Remember that rate and time are inversely proportional to each other.
- **R1 * T1 = R2 * T2 → R1/R2 = T2/T1**
- Example: 4 monkeys take 6 hours to tear down a house. How long will it take 7 monkeys to tear down the house if they work at the same rate?
- 4/7 = X/6
- X = 24/7 hours

	Rate	Time	Work
Case 1			
Case2			

Ratios

- Ratio between two number x and y is written as x:y. It can also be written as $\frac{x}{y}$
- The equality of two ratios is called proportion. a,b,c and d are in proportion a:b::c:d
- In proportion, product of extremes = product of means
- i.e., a*d=b*c
- Multiple ratios: for example, X:Y = 4:5 and Y:Z = 10:15 the X:Y:Z=8:10:15
- **Mixture problems**
- **VolX*ConcentrationX + VolY*ConcentrationY = VolXY*ConcentrationXY**
- **Organize information in a data table**
- Columns: Original Solution, [Solution Removed], Solution Added, Final Solution
- Rows: Concentration, Volume, Product (i.e., Contratration*Volume)

Combinatorics- Permutation

- Permutation if the number of available items and spots are equal

$$nP_r = n!$$

- Permutation in a circular arrangement

$$nP_r = (n-1)!$$

- Permutations Formula if the **n** items are to be arranged in **k** spots

$$nP_r = \frac{n!}{(n-k)!}$$

- Permutations with repeating elements
- If the repeating elements are a and b then the formula becomes

$$nP_r = \frac{n!}{(a)!(b)!...}$$

- For example, the following number of ways “challenge” can be arranged in a row as e and l are repeated 2 times

$$\frac{9!}{(2)!(2)!}$$

Combinatorics - Combination

- Combination formula

$$nC_r = \frac{n!}{k! * (n-k)!}$$

- **Binomial Probability**: nCk * pk * qn-k
- n = number trials
- k = number of successes
- n – k = number of failures
- p = probability of success
- q = probability of failure
- p + q = 1

Probability

- Mutually exclusive events – two events are called mutually exclusive if they can never occur together.
- Example, getting a 770 and a 700 on the same GMAT test
- P(A and B) = 0

- Complementary events – events are called complementary if one and only one of them must occur.
- Example, getting into Harvard Business school or not getting into Harvard Business School
- P(A or B) = 1

- Independent events – events are independent if the occurrence of one event does not affect the probability of the occurrence of another.
- Example, Flips of a coin, picking balls out of a jar with replacement

- Dependent events – if the occurrence of one event affects the probability of another.
- Example, picking balls out of a jar without replacement

- Probability of one event and another
- Independent events – when determining the probability of multiple independent events, simply multiply the individual probabilities.
- P(A and B) = P(A)*P(B)

Probability contd

- Probability of one event or another
- Example, probability of winning a lottery and the probability of getting 800 in GMAT
- P(A or B) = P(A)+P(B) – P(A and B)

- Probability of a single event

$$P(A) = \frac{Number - of - favourable - outcomes}{Total - number - of - outcomes}$$

Probability examples

- Basic Examples: You have one red marble and 3 white marbles.
- If you draw two marbles **without replacement**, what is the probability that you will have one red and white?
  - Method 1: (1/4 * 3/3) + (3/4 + 1/3) = 1/2
  - Method 2: (1C1 *3C1)/4C2 = 1/2
- If you draw two marbles **with replacement**, what is the probability that you will have one red and white? The easiest way to solve this problem is to consider the two possibilities: (i) you draw a red then a white, and (ii) you draw a white then a red.
  - (1/4 * 3/4) + (3/4 + 1/4) = 3/8

- Ordered Set Probability Example: 5 girls, 3 boys to be lined up in a row.
- Each person and each position is considered to be distinct.
- Ways to order the children: 8!
- Ways to order so that there is a girl on each end: 5 * 4 * 6! or (5C1 * 4C1 * 6!)
- Probability that there is a girl on each end: (5C1 * 6! * 4C1)/8!

- Unordered Set Probability Example: 11 marbles: 5 black, 4 white, 2 yellow. 5 marbles are picked out.
- Ways to pick 5 marbles: 11C5
- Ways to pick 5 black: 5C5
- Ways to pick 3 black and 2 white: 5C3 *4C2
- Probability of picking 5 black: 5C5/11C5
- Picking 3 black and 2 white: (5C3 *4C2)/11C5

Probability examples contd

- Probability of picking 2 queens from a deck of cards.
- Ways to pick two cards = 52C2
- Ways to pick two queens = 4C2
- Probability: 4C2/52C2
- 2 freshman, 2 sophomores, 2 juniors and 2 seniors. If a 2-person group must consist of persons from different classes, what is the probability of choosing a group consisting of one freshman and one sophomore?
- # of possible 2-person groups: 4C2 * 2 * 2 = 24
- Ways to choose a freshman/sophomore group: 2C1 *2C1 = 4
- Probability: 4/24 = 1/6

- Bill has a small deck of 12 playing cards made up of only 2 suits of 6 cards each. Each of the 6 cards within a suit has a different value from 1 to 6; thus, there are 2 cards in the deck that have the same value. Bill likes to play a game in which he shuffles the deck, turns over 4 cards, and looks for pairs of cards that have the same value. What is the chance that Bill finds at least one pair of cards that have the same value?
- 1 – P(No Pairs)
- Two ways to calculate P(No Pairs)
  - Select each card one at a time: (12/12)(10/11)(8/10)(6/9) = 16/33
  - Use combinations:
    - 6C4 = Number of ways to choose 4 numerical values
    - 2 = Number of cards at each value
    - 12C4 = Number of ways to choose 4 cards from 12
    - (6C4 * 2 * 2 * 2 * 2)/ 12C4 = 16/33
  - 1 – 16/33 = 17/33
- Alternative method for calculating the probability of No Pairs (i.e., assume you draw serially and find the probability for each of the four draws given the “no pairs” restriction—i.e., on the first draw, all 12 cards are candidates, on the second draw, all remaining 11 cards are candidates except 1 card, etc.):
  - (12/12) * (10/11) * (8/10) * (6/9) = 16/33
- **Note: The problems above can also be done by drawing the cards serially and multiplying the individual probabilities together.**

Statistics

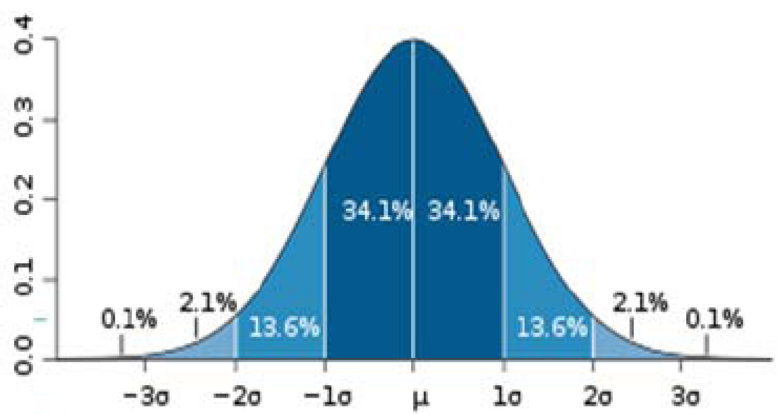
- Average or arithmetic mean of the set is given by the following formula
- $Average = \frac{Sum}{Number - of - terms}$
- So, Average * Number of terms = Sum
- Median is the middle value in a set
- Mode is the most frequent element in a set
- Range is the difference between the highest and lowest element in a set
- For example, {-10,-3,-3,0,3,5,6,7}
- Median= $\frac{0+3}{2}$
- Mode = -3
- Range = 7-(-10)=17
- **Median and Mean**
- In a evenly spaced set, median and mean will be equal

Statistics

- **Standard deviation** can be found using the following steps
- Find the mean
- Compute the differences between mean and each number
- Square the differences and add them together
- Divide the sum of the squared differences by the number of terms
- Take the square root of the result
- Example,

Result	Mean	Difference	Diff. Squared
2	5	-3	9
4	5	-1	1
4	5	-1	1
4	5	-1	1
5	5	0	0
5	5	0	0
7	5	2	4
9	5	4	16

- Average squared diff. = 32/8 = 4
- Standard deviation = $\sqrt{4}$ = 2



A plot of a normal distribution (or bell curve). Each colored band *has* a width of one standard deviation.

Statistics- Weighted Average

$$\text{Weighted avg} = \frac{w_1x_1+w_2x_2+w_3x_3+\ldots+w_nx_n}{w_1+w_2+\ldots\ldots+w_n}$$

Example problem:

Are at least 10 percent of the people in Country X who are 65 years old or older employed?

(1) In Country X, 11.3% of the population is 65 years old or older
(2) In Country X, of the population 65 years old or older, 20 percent of the men and 10 percent of the women are employed.

Question: is at least 10% of some particular group of people employed?

(1) This particular group composes 11.3% of total population. Clearly insufficient, as no info about employment rates in this group.

(2) 10% of men in this group and 20% of women in this group are employed. No matter how many men and women are in this group, more than 10% will be employed. This is because the weighted average of 2 individual averages (10% and 20%) must lie between these individual averages, so percent of employed people in this group is between 10% and 20%. Sufficient.

Answer: B.

Statistics- Weighted Average- Another example

In a corporation, 50 percent of the male employees and 40 percent of the female employees are at least 35 years old. If 42 percent of all the employees are at least 35 years old, what fraction of the employees in the corporation are females?

A. 3/5

B. 2/3

C. 3/4

D. 4/5

E. 5/6

Let's say that "x" is the % of male employees in the corporation. So, (1-x) is the % of female employees in the same company.

Therefore: 50%(x)+40%(1-x)= 42%

(1/2)x + (2/5) (1-x) = 21/50

So, x=1/5

Then 1-x=4/5

Answer D

Sets

- **Use a Double Set Matrix** (i.e., use a 3-by-3 grid)
- Good when you have two attributes, each with two values
- Third row/column is for "Total"
- E.g., persons at a party are (i) either M or F and (ii) either Smart or Stupid
- **AuB = A + B – (AnB)**
- **If there are three sets A, B, and C, then**:
- Total number of people/Number of people in at least one set:[1]
  - AuBuC = A + B + C – AnB – AnC – BnC + AnBnC
- Number of people in exactly one set:
  - A + B + C – 2*AnB – 2*AnC – 2*BnC + 3*AnBnC
- Number of people in exactly two of the sets"
  - AnB + AnC+ BnC – 3*AnBnC
- Number of people in two or three sets:
  - AnB + AnC + BnC – 2*AnBnC
- Number of people in exactly three of the sets =
  - AnBnC
-

[1] This is the only formula worth remembering.

Fractions

- (a/b)/(c/d) = (a/b) * (d/c)
- Comparing Fractions
  - Technique one: Cross multiply.
  - Technique two: Find the LCD and compare the size of the numerators.
    - Example: What is bigger 3/8! or 3/(5*7!)
      - The LCD is (5*8!), so multiply the first expression by 5/5 and the second expression by 8/8.

Percents

<u>“Percentage change” and “percentage increase” are NOT the same as “percentage of”</u>

Percent change/increase = (New - Old)/Old

Percentage of = New/Old

5 is what percent of 2?

$5 = (X/100)*2$

$5/2 * 100 = 250\%$

5 is what percent greater than 2?

$(5-2)/2 * 100 = 150\%$

Basic equations

- Determining whether 2 equations involving 2 variables will be sufficient to solve for all the variables.
- Sufficient if both of the equations are (i) linear (i.e., no x2, y2, xy or x/y terms) and (ii) unique.
- Remember that you can solve by either:
  - Substitution or
  - Combination
    - Adding/subtracting the equations,
    - Multiplying/dividing the equations, or
    - Adding the same value to both sides of both equations (this technique is only needed for manipulation purposes).
- **Step-by-Step Solving Method**:
  - Isolate the expression within the absolute value brackets so that it appears by itself on one side of the equation (i.e., $|x| = a$)
  - Solve the equation for two cases:
    - $x = a$
    - $x = -a$
      - If the problem involves an equality sign, you will need to flip the inequality sign at this step!
        - » $|x| < 7$; the two cases are $x < 7$ and $x > -7$ → $-7 < x < 7$
  - Plug your solutions back into the original equation to check validity (<u>this is not an optional step</u>!).
- **Absolution Value Expressions Contained in Inequalities**
  - If the arrow is pointing to the left, you will have an "and" solution
    - $|x| < 3$ → $-3 < x < 3$
  - If the arrive is pointing to the right, you will have an "or" solution
    - $|x| > 3$ → $x < -3$ or $x > 3$

Equations with Exponents

- Equations w/ one or more variables raised to an even power: Generally 2 solutions.
  - Need to be careful when appearing in DS problems.
- Equations w/ all variables raised to an odd power: Often 1 solution.
- Rewrite exponential equations so that the components have either the same base or same exponents.
- If there is a variable in the base, be sure to consider -1, 0 and 1.
- Eliminating Roots
  - Methodology:
    - Square both sides of the equation.
    - Solve for X.
    - Check solution(s) by plugging them back in.
  - Note: No need to plug the solution back in if you raise the sides of the equation to an odd power.

Quadratic Equations

- **Quadratic Formula**
- In an equation like ax2 + bx + c = 0
- You can solve for x using the Quadratic Formula:

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- For example, $x^2 + 5x + 6$
- To factor the quadratic equation you need to find two integers whose product is 6 and whose sum is 5.
  (x+3)(x+2)

- **Formulae**

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

$$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$$

$$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$$

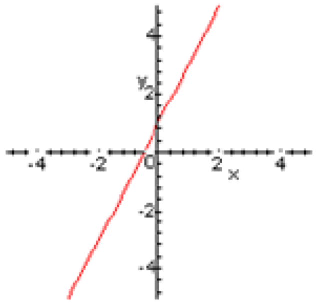
$$a^2 - b^2 = (a+b)(a-b)$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Functions

- A function assigns a unique value to each input of a specified type. $f(x) = (x^2 + 5)$
- For example,
- Then if the question asks what is the value of f(2) then just substitute 2 for all the occurrence of x
- $f(2) = (2^2 + 5) = 9$
- Functions can be visualized by graphing it in the coordinate plane
- f(x)=2x+1



Input	Output	(x,y)
-2	f(-2)=2(-2)+1	(-2,-3)
-1	f(-1)=2(-1)+1	(-1,-1)
0	f(0)=0+1	(0,1)
1	f(1)=2(1)+1	(1,3)
2	f(2)=2(2)+1	5

Inequalities

Negative signs and Division result in flipping the symbol		
Operation	**Example**	**Result**
Addition	8+LT2	LT10
Subtraction	8-LT2	GT6
Multiplication	1) 8 * LT2 2) -2* LT2	1) LT16 2) GT(-16)
Division	1) . $\frac{8}{LT2}$ 2) . $\frac{-8}{LT2}$	1) GT4 2) LT(-4)

Inequalities contd.

- XY > 0 - Means X and Y have same sign
- XY < 0 - Means X and Y have different sign
- $X^2 - X < 0$ = $X(X-1) < 0$ - Means X is between 0 and 1, 0<X<1
- $X^2 - X > 0$ = $X(X-1) > 0$ - Means either X <-1 or X > 1
- .$\sqrt{X} > X$ - Means 0<X<1
- .$\sqrt{X} < X$ - Means X>1
- . $|x| > x$ - Means x is a negative number
-

Rules of inequalities

- Rule 1: if x>y, then x-y>0 and x=y+k; where k>0
- For example: 3> 2. Therefore, 3-2=1>0 and 3=2+1, where 1>0

- Rule 2: if x<y, there x-y < 0 and x=y-k; where k>0
- For example: 2<3 so, 2-3=-1<0 and 2=3-1, where 1>0 or 2=3-1 where -1<0

- Rule 3: if x>y and k is a real number, x+k>y+k, irrespective of the nature of k.
- For example, 20>10 and k=2
- 20+2>10+2 => 22> 12.
- Alternatively k=-40; 20-40> 10-40 => -20 > -30

- Rule 4: if x>y and k>0, then kx>ky and $\frac{x}{k} > \frac{y}{k}$
- For example, 20>10, and k=2, then 20*2>10*2

- Rule 5: If x>y and k<0, then kx<ky and $\frac{x}{k} < \frac{y}{k}$
- For example, 20>10, and k=-2 then 20*(-2)<10*(-2)

- Rule 6: if x>y and x>0 and y>0, then $\frac{1}{x} < \frac{1}{y}$

Inequalities

- Sample question
- Is k greater than 3?

  (1) (k - 3)(k- 2)(k - 1) > 0
  (2) k > 1

- **Is K>3 ?**

  (1) $(k-3)(k-2)(k-1)>0$

  The product of 3 numbers is positive if all three are positive (+++) OR two of them are negative and the third one is positive (+--).

  Note that: out of 3 numbers (k-3) is the least one and (k-1) is the biggest one.

  (+)(+)(+) is when even the least one is positive so when k-3>0 --> k>3;
  (+)(-)(-) is when the biggest one is positive (k-1>0 -->k>1) and the next one (hence the least one too) negative (k-2<0 -->k<2), so when 1<k<2;

  So (k - 3)(k- 2)(k - 1) > 0 means that:k>3 or 1<k<2 --> k may or may not be more than 3. Not sufficient.

  (2) k>1 . Clearly insufficient.

  (1)+(2) Intersection of the ranges from (1) and (2) is the range we had in (1) k>3 or 1<k<2 , so k may or may not be more than 3. Not sufficient.

  Answer: E.

Sequences and Progressions

- Arithmetic Progression
- A sequence of numbers is said to be in arithmetic progression when the difference between two consecutive terms is a constant.
- a-first term, d- common difference of the A.P. then the terms of the AP can be represented as a,a+d,a+2d,a+3d….. Then nth term can be found by

$$t_n = a + (n-1)d$$

- Sum S_n of the first n terms can be represented as

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

- If first term and last term is known

$$S_n = \frac{n}{2}[first - term + last - term]$$

- Sum of n odd numbers = n^2
- Sum of first n even numbers = n(n+1)

References

- GMAT club website
- Wiki online
- www.onlinemathlearning.com
- Slingfox notes
- Several other online pages